Magnetopolaron effects in light reflection and absorption of a three-level system in a QW

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(February 1, 2008)

The light absorption $\mathcal{A}$ and reflection $\mathcal{R}$ coefficients for a three-level system in a quantum well (QW) in a strong magnetic field **H**, directed perpendicularly to the QW's plane, have been calculated. The energy levels correspond to the ground state and to magnetopolaron energy levels $\hbar\omega_{1(2)}$ with the inverse lifetimes $\gamma_{r1(2)}$ and $\gamma_{1(2)}$. The inverse radiative lifetime of an EHP $\gamma_{r\xi}(\mathcal{K}_\perp) \neq \mathbf{0}$ only under condition $\mathcal{K}_\perp \leq \omega_\xi n/c$, where $\hbar\omega_\xi$ is the EHP energy. It has been shown that $\gamma_{r\xi}(\mathcal{K}_\perp)$ is proportional to H. The values $\gamma_{ra}(\mathcal{K}_\perp)$ and $\gamma_{rb}(\mathcal{K}_\perp)$ for an excitation consisting of a hole and a usual magnetopolaron have been calculated. The index $a(b)$ designates a magnetopolaron upper (lower) term. In the resonance at $H_{res} = m_e c\omega_{LO}/|e|$ $\gamma_{ra}^{res} = \gamma_{rb}^{res} = \gamma_{r\xi_0}^{res}/2$, where the index ξ_0 corresponds to the EHP with $n_e = n_h = 1$. γ_{ra} and γ_{rb} are strongly dependent on $H - H_{res}$. A numerical estimate for GaAs is: $\gamma_{r\xi_0}^{res} = \gamma_0 = 5.35 \cdot 10^{-5} eV$. The dependencies $\mathcal{A}(H)$ and $\mathcal{R}(H)$ for various values of ω_l in the magnetophonon resonance vicinity have been obtained. The existence of "combined"and "weak" polarons has been predicted. The resonant value H_{res} for the combined polaron case depends on a QW's depth and width.

I. INTRODUCTION

When a polaron state is formed in a magnetic field the role of the electron-phonon interaction grows sharply under the resonant condition

$$\omega_{LO} = j\Omega, \quad j = 1, 2, 3, ..., \tag{1}$$

ω_{LO} is the LO phonon frequency, Ω is the electron (hole) cyclotron frequency. The intersections of the electron-phonon system energy levels as functions of magnetic field stem (see Fig.1). A transition to the magnetopolaron states results in energy levels splitting in crossing points. For the first time magnetopolaron energy level splitting was discovered in light absorption in bulk $InSb$. [1]

Polaron state formation takes place in both three (3D) and two dimensional (2D) semiconducting systems. These states are very important in frequency dependence formation of magnetooptical effects, such as reflection, interband absorption, cyclotron resonance and Raman scattering (see, for instance, the reviews Refs. [2,3,4]). The systems distinguish by the electron (hole) spectrum: in 3D systems these are one-dimensional Landau bands , whereas in 2D structures these are discrete energy levels. This difference results in different splitting of the energy levels of the electron-phonon system: In 3D systems it is the value of order $\alpha^{2/3}\hbar\omega_{LO}$, [5] whereas in 2D structures it is the value of order $\alpha^{1/2}\hbar\omega_{LO}$, [6,7,8,9] where α is Fröhlich's non-dimensional electron-LO phonon coupling constant. [10]

A perfect single QW of a finite depth is considered below as an example of a 2D system. The inhomogeneous broadening effects of the energy levels are ignored. We shall calculate the non-dimensional reflection coefficient $\mathcal{R}$ and the non-dimensional light absorption coefficient $\mathcal{A}$ which are determined as the relation of the reflected or absorbed light flux to the incident one. There are both the EHP discrete energy levels and the continuum spectrum. The optical effects are due to the resonance of the energies of incident light $\hbar\omega_l$ and the EHP or magnetopolaron discrete energy levels. Far from the H and ω_l values, corresponding to the magnetopolaron resonance, we will consider the system as a *two-level* one (the first level is the crystal ground state energy and the second is the EHP energy level). In the vicinity of the magnetopolaron resonance we consider the system as a *three-level* one, including the ground state energy and two polaron levels.

Light absorption and reflection in quasi-2D systems had been continually considered earlier. In calculations of absorption usually the low order perturbation theory on the light-electron system interaction (see, for instance, Refs. [11,12,13]) had been used. The result for the light absorption coefficient contains the factor $\Delta_{\gamma_\rho}(\hbar\omega_l - E_\rho)$, where E_ρ is the electron excitation energy measured from the ground state energy,

$$\Delta_\gamma(E) = \frac{1}{\pi}\frac{\hbar\gamma/2}{E^2 + (\hbar\gamma/2)^2} \tag{2}$$

is the function transiting into the Dirac δ - function when $\gamma \to 0$, γ_ρ is the non-radiative inverse lifetime

on the energy level ρ. This result implicates some self-contradiction: The value $\Delta_\gamma(E) \to \infty$ at $E = 0$ and $\gamma \to 0$, whereas the non-dimensional absorption coefficient $\mathcal{A}$ cannot exceed the value of one. This self-contradiction is lifted if one goes out the perturbation theory limits on the light-electron coupling constant, i. e. the contributions of all the orders on this coupling constant are summarized. This summarizing means taking into account the sequence of all the absorption and reradiation processes of the light quantum $\hbar\omega_l$. There appears a new concept of the radiation lifetime $\gamma_{r\rho}^{-1}$ of an electronic excitation which has been introduced firstly for the excitons in a QW at $H = 0$. [14]

At first, the new approach had been used to describe light reflection from QW [12,14,15,16] at frequencies ω_l close to an exciton energy, then an appropriate theory for light absorption [17,18] has been created. It has been shown that the previous results obtained with the help of the perturbation theory are applicable under condition

$$\gamma_{r\rho} << \gamma_\rho. \tag{3}$$

The new theory has to be used to describe the features of light reflection and absorption by ideal QWs in magnetic fields because the non-radiative values γ_ρ for the discrete energy levels of EHPs are small and the condition of Eq. (3) may be unfeasible. We will calculate the coefficients $\mathcal{R}$ and $\mathcal{A}$ for the case of the incident light perpendicular to the QW plane, taking into account the magnetopolaron effects.

The paper is organized as follows. The magnetopolaron classification is described in Seq. II, the expressions for the electric fields left and right from a QW at the normal irradiation by the incident light in the case of the multilevel system are obtained in Seq. III. The formulae for light reflection and absorption in a QW are calculated in Seq. IY. In Seq. Y and Seq. YI, the inverse radiative lifetime for an EHP in QW in magnetic field and the inverse radiative lifetimes of two magnetopolaron states are calculated, respectively. The inverse non-radiative lifetimes of the magnetopolaron states are calculated in Seq. YII. The numerical calculation results for light absorption and reflection are given in Seq. YIII.

II. MAGNETOPOLARON CLASSIFICATION

Fig. 1 shows (the full lines) the non-dimensional electron-phonon system energy levels $E/\hbar\omega_{LO}$ in a QW, pertaining to the size-quantization quantum number l, as functions of $\Omega/\omega_{LO} = j$,

$$\Omega = |e|H/m_{e(h)}c \tag{4}$$

is the cyclotron frequency, e is the electron charge, $m_{e(h)}$ is the electron (hole) effective mass. E is measured from the energy $\varepsilon_l^{e(h)}$ corresponding to the l size-quantization energy level. It is supposed that all the phonons relevant to the magnetopolaron creation have the same non-dispersal frequency ω_{LO}.

The polaron states correspond to the crossing points. The "double" polarons are denoted with the black circles and appropriate to crossings of two terms only. Let us consider some crossing point corresponding to the number j (see Eq. (1)). n is the Landau quantum number of the energy level coming through this crossing point, the phonon number $N = 0$ (see Fig. 1). Then the condition of the double polaron existence is

$$2j > n \geq j. \tag{5}$$

It is easy to note that the only double polaron A corresponds to $j = 1$. Two double polarons D and E correspond to $j = 2$, three double polarons F, K and L correspond to $j = 3$ and so on. The polarons on the left of the value $\Omega/\omega_{LO} = 1/3$ are not noted in Fig.1. The triple polarons (corresponding to three terms crossings) are above the double polarons, the quateron polarons are above the triple polarons and so on. There are j polarons of every sort at the given j. For the first time the triple polarons in bulk semiconductors and in QWs have been considered in Ref. [19] and in Refs. [20,21,22], respectively.

Note that the Landau level's equidistancy is needed for three and more levels crossings. The triple polaron theory [22] is relevant to the case when the energy corrections due to the band non-parabolicity or excitonic effects are smaller than the triple polaron splitting. But in the case of the double polarons, the equidistancy violation is not an obstacle because crossing of two terms exists at any rate.

All the above mentioned polarons correspond to the integer values of j. At Fig. 1 there are some other crossings of the terms with the size-quantization quantum number l (solid lines) marked with empty circles. They are relevant to the fractional values of j. Because these crossings are characterized by the values $\Delta N \geq 2$, the real direct transitions with emitting of *one* LO phonon are impossible. We call such polarons *weak* ones. To calculate splittings, one has to take into account the virtual transitions between the crossing terms or two-phonon interaction. Splittings of the weak polarons ΔE_w are much smaller than for the polarons with integer j. The contributions of the intermediate transitions into the value ΔE_w are of higher order on α than $\alpha^{1/2}$.

The crossing depiction becomes much more complicated when two or more values of the size-quantization quantum numbers l are taken into account.Besides the ordinary polarons corresponding to the energy level l' (for example the polaron A') the "combined" polarons [13] appear where electron-phonon interaction bounds two electron energy levels with different Landau quantum numbers n and different l (or with different l and

the same n). Two terms corresponding to the quantum number l' (dotted lines) and the combined polaron P and Q positions (tilted crosses) are shown in Fig. 1. The combined weak polaron is marked with R.

The combined polarons have an interesting feature: The corresponding resonant magnetic field values depend on the distance $\Delta\varepsilon = \varepsilon_{l'} - \varepsilon_l$ between the size-quantization energy levels l and l' and consequently on the QW's depth and width. With the help of Fig. 1 one can obtain easily

$$\left(\frac{\Omega}{\omega_{LO}}\right)_P = 1 - \frac{\Delta\varepsilon}{\hbar\omega_{LO}}, \quad \left(\frac{\Omega}{\omega_{LO}}\right)_Q = 1 + \frac{\Delta\varepsilon}{\hbar\omega_{LO}}. \quad (6)$$

One more combined polaron sort [13] exists under condition

$$\Delta\varepsilon = \hbar\omega_{LO}, \quad (7)$$

when, for instance, the terms $l', n = 0, N = 0$ and $l, n = 0, N = 1$ coincide at any magnetic field values. The definite separation between the energy levels l and l' is necessary to satisfy the condition (7). This could be done by choosing the QW's depth and width.

The situation depicted in Fig. 1 is applicable when the energy separations between the neighbor size-quantization levels l, $l-1$, $l+1$ are much more than the values ΔE of polaron splittings. So long as the separations diminish with growing of the QW's width, some restriction is implicated on the value d from above (see the numerical estimates in Ref. [23]).

III. ELECTRIC FIELDS OUT OF QW

Let us suppose that an electromagnetic (EM) wave (or some superposition of EM waves, corresponding to a light impulse) falls perpendicular on a single QW from the left. The electric field intensity is *

$$E_{0\alpha}(z,t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} d\omega e^{-i\omega(t-zn/c)}\mathcal{E}_{0\alpha}(\omega) + c.c., \quad (8)$$

where n is the barrier refraction index.

Let us introduce the designations

$$\mathcal{E}_{0\alpha}(\omega) = 2\pi E_0 e_{l\alpha}\mathcal{D}_0(\omega), \quad (9)$$

where E_0 is the scalar amplitude, $\mathbf{e}_l$ is the incident light polarization vector ; $\mathcal{D}_0(\omega)$ is a frequency function;

$$\mathcal{D}_0(\omega) = \delta(\omega - \omega_l) \quad (10)$$

for monochromatic light of frequency ω_l.

It is supposed that the incident EM wave has a circular polarization

$$\mathbf{e}_l = \frac{1}{\sqrt{2}}(\mathbf{e}_x \pm i\mathbf{e}_y), \quad (11)$$

where $\mathbf{e}_x, \mathbf{e}_y$ are the unit vectors along the x and y axis, respectively.

The incident EM wave generates the excited electronic states with the energies characterized by the index ρ. For instance, in a magnetic field the index ρ for the EHP includes $l_e, l_h, n_e = n_h$. For the infinitely deep QW $l_e = l_h$. The state ρ is characterized by the energy $\hbar\omega_\rho$, which is measured from the ground state energy, by the inverse radiative lifetime $\gamma_{r\rho}$ and by the inverse non-radiative lifetime γ_ρ. Let us consider the QW with $d << c/\omega_l n$.

Then the electric field intensities $\mathbf{E}_{left(right)}(z,t)$ on the left and right of the QW are determined as [24]

$$\mathbf{E}_{left(right)}(z,t) = \mathbf{E}_0(z,t) + \Delta\mathbf{E}_{left(right)}(z,t), \quad (12)$$

$$\Delta E_{\alpha\ left(right)}(z,t) = E_0 e_{l\alpha}\int_{-\infty}^{\infty} d\omega e^{-i\omega(t\pm zn/c)}\mathcal{D}(\omega) + \\ + c.c., \quad (13)$$

where the upper (lower) sign is relevant to the index $left$ ($right$). The frequency partitition is determined as †

$$\mathcal{D}(\omega) = -\frac{4\pi\chi(\omega)\mathcal{D}_0(\omega)}{1 + 4\pi\chi(\omega)}, \quad (14)$$

$$\chi(\omega) = \frac{i}{4\pi}\sum_\rho \frac{\gamma_{r\rho}}{2}[(\omega - \omega_\rho + i\gamma_\rho/2)^{-1} + \\ + (\omega + \omega_\rho + i\gamma_\rho/2)^{-1}]. \quad (15)$$

Eqs. (12)–(15) determine the electric fields on the both sides of the QW and, consequently, the values of the energy fluxes in transmitted and reflected light for the multilevel system. The second term in the square brackets in the RHS of Eq. (15) is non-resonant, but it is necessary to satisfy the relation

$$\chi^*(\omega) = \chi(-\omega). \quad (16)$$

In a real QW of a finite depth in magnetic field, there are always discrete energy levels (one at least) together with the continuous spectrum. Thus, the index ρ in the RHS

*One can remove the complex conjugate term and introduce (instead of $\mathcal{E}_{0\alpha}(\omega)$)

$$E_{0\alpha}(\omega) = \mathcal{E}_{0\alpha}(\omega) + \mathcal{E}^*_{0\alpha}(-\omega) = 2\pi E_0[e_{l\alpha}D_0(\omega) + e^*_{l\alpha}D^*_0(-\omega)].$$

†It is supposed in Eq. (15) that both circular polarizations corresponds to the excitation of one of two types of EHPs the energies of which are equal each other (see Eq. (88) below).

of Eq. (15) takes both discrete and continuous values. Sometimes in a magnetic field, the large level number has to be taken into account (see [25] where the results of an excitation with an asymmetrical light impulse have been investigated). Discussing a two-level system we mean a situation when all the energy levels can be neglected, but one which is resonant with $\hbar\omega_l$.

The absorption and reflection coefficients for a two-level system with one excitonic energy level at $H = 0$ have been calculated in Refs. [14,15,16,17,18]. The electric field intensities are expressed as

$$E_{\alpha\ left} = E_0 e_{l\alpha} \left\{ e^{-i\omega_l(t-zn/c)} - \right. \\ \left. - \frac{i\gamma_r}{2(\omega_l - \omega + i\Gamma/2)} e^{-i\omega_l(t+zn/c)} \right\} + c.c., \quad (17)$$

$$E_{\alpha\ right} = E_0 e_{l\alpha} e^{-i\omega_l(t-zn/c)} \times \\ \times \left\{ 1 - \frac{i\gamma_r}{2(\omega_l - \omega + i\Gamma/2)} \right\} + c.c., \quad (18)$$

$$\Gamma = \gamma + \gamma_r. \quad (19)$$

One can obtain Eq. (17) and Eq. (18) with the help of Eqs. (10), (12)–(15) if one term in the sum on ρ in the RHS of Eq. (15) is preserved using the designations

$$\omega_\rho = \omega, \qquad \gamma_{r\rho} = \gamma_r, \qquad \gamma_\rho = \gamma, \quad (20)$$

and neglecting the second non-resonant term in the square brackets in the RHS of Eq. (15).

The three-level system consists of the ground state of an electronic system and two energy levels $\hbar\omega_{1(2)}$ with the inverse radiative $\gamma_{r1(2)}$ and non-radiative $\gamma_{1(2)}$ lifetimes. The calculation of the electric field intensities for the three-level system is a little more complicated because one has to solve the square equation (see Eq. (23) below). Applying Eqs. (10), (12)–(15) and omitting the non-resonant term in the RHS of Eq. (15) one obtains

$$E_{\ \alpha\ left}(z,t) = E_0 e_{l\alpha} \left\{ e^{-i\omega_l(t-zn/c)} - ie^{-i\omega_l(t+zn/c)} \times \right. \\ \left. \times \left[\frac{\bar\gamma_{r1}}{2(\omega_l - \Omega_1 + iG_1/2)} + \frac{\bar\gamma_{r2}}{2(\omega_l - \Omega_2 + iG_2/2)} \right] \right\} + \\ + c.c., \quad (21)$$

$$E_{\ \alpha\ right}(z,t) = E_0 e_{l\alpha} e^{-i\omega_l(t-zn/c)} \times \\ \times \left\{ 1 - i \left[\frac{\bar\gamma_{r1}}{2(\omega_l - \Omega_1 + iG_1/2)} + \frac{\bar\gamma_{r2}}{2(\omega_l - \Omega_2 + iG_2/2)} \right] \right\} \\ + c.c.. \quad (22)$$

The complex quantities $O_1 = \Omega_1 - iG_1/2$ and $O_2 = \Omega_2 - iG_2/2$ ($\Omega_{1(2)}$ and $G_{1(2)}$ are real on their definition) are the precise solutions of the equation

$$(O - \omega_1 + i\gamma_1/2)(O - \omega_2 + i\gamma_2/2) + \\ + i\frac{\gamma_{r1}}{2}(O - \omega_2 + i\gamma_2/2) + \\ + i\frac{\gamma_{r2}}{2}(O - \omega_1 + i\gamma_1/2) = 0, \quad (23)$$

which are equal

$$(\Omega - iG/2)_{1,2} = \frac{1}{2}\left[\tilde\omega_1 + \tilde\omega_2 \pm \sqrt{(\tilde\omega_1 - \tilde\omega_2)^2 - \gamma_{r1}\gamma_{r2}} \right], \quad (24)$$

where

$$\tilde\omega_{1(2)} = \omega_{1(2)} - i\Gamma_{1(2)}/2, \quad \Gamma_{1(2)} = \gamma_{1(2)} + \gamma_{r1(2)}. \quad (25)$$

Note that the value in the square root in Eq. (24) is, generally speaking, a complex one. In the RHS of Eq. (24) the upper (lower) sign corresponds to the index 1(2). The following designations are used

$$\bar\gamma_{r1} = \gamma_{r1} + \Delta\gamma, \qquad \bar\gamma_{r2} = \gamma_{r2} - \Delta\gamma, \quad (26)$$

$$\Delta\gamma = \frac{\gamma_{r1}[\Omega_2 - \omega_2 - i(G_2 - \gamma_2)/2]}{\Omega_1 - \Omega_2 + i(G_2 - G_1)/2} + \\ + \frac{\gamma_{r2}[\Omega_1 - \omega_1 - i(G_1 - \gamma_1)/2]}{\Omega_1 - \Omega_2 + i(G_2 - G_1)/2}. \quad (27)$$

Comparing Eq. (21) and Eq. (22) with Eq. (17) and Eq. (18) for the two-level system, one can see that the addition of the contributions from the energy levels 1 and 2 is accompanied by the substitutions: $\Omega_{1(2)}$ instead of $\omega_{1(2)}$, $G_{1(2)}$ instead of $\Gamma_{1(2)}$ and $\bar\gamma_{r1(2)}$ instead of $\gamma_{r1(2)}$.

To calculate the electric field intensities for the four-level system one has to solve a third order equation and so on. It is impossible to solve this task precisely for an arbitrary number ρ of the energy levels. However, in the case

$$\gamma_{r\rho} << \gamma_\rho, \quad (28)$$

when the perturbation theory on light-electron system interaction is applicable (what results in the possibility of neglecting the term $4\pi\chi(\omega)$ in the denominator in the RHS of Eq. (14)), we obtain the result for the arbitrary number of the energy levels

$$E^{\ many}_{\alpha\ left}(z,t) = E_0 e_{l\alpha} \left\{ e^{-i\omega_l(t-zn/c)} - \right. \\ \left. - ie^{-i\omega_l(t+zn/c)} \sum_\rho \frac{\gamma_{r\rho}}{2(\omega_l - \omega_\rho + i\gamma_\rho/2)} \right\} + c.c., \quad (29)$$

$$E^{\ many}_{\alpha\ right}(z,t) = E_0 e_{l\alpha} e^{-i\omega_l(t-zn/c)} \times \\ \times \left[1 - i\sum_\rho \frac{\gamma_{r\rho}}{2(\omega_l - \omega_\rho + i\gamma_\rho/2)} \right] + c.c., \quad (30)$$

IV. LIGHT REFLECTION AND ABSORPTION COEFFICIENTS.

Having the electric field intensities out of the QW we can calculate the reflection and absorption coefficients of light. The Umov-Poynting vector $\mathbf{S}_{left}$ on the left of the QW is

$$\mathbf{S}_{left} = \mathbf{S}_0 + \Delta\mathbf{S}_{left}, \tag{31}$$

where

$$\mathbf{S}_0 = \frac{c}{2\pi}E_0^2\mathbf{e_z} \tag{32}$$

is the incident light flux, $\Delta\mathbf{S}_{left}$ is the reflected light flux directed along $-\mathbf{e}_z$. The light reflection coefficient is determined as

$$\mathcal{R} = \frac{|\Delta\mathbf{S}_{left}|}{|\mathbf{S}_0|}, \tag{33}$$

the non-dimensional light absorption coefficient is determined as

$$\mathcal{A} = \frac{|\mathbf{S}_{left} - \mathbf{S}_{right}|}{|\mathbf{S}_0|}, \tag{34}$$

and the light transition coefficient is

$$\mathcal{T} = 1 - \mathcal{R} - \mathcal{A} = \frac{|\mathbf{S}_{right}|}{|\mathbf{S}_0|}. \tag{35}$$

With the help of Eq. (17) and Eq. (18) we obtain the results for the two-level system

$$\mathcal{R} = \frac{(\gamma_r/2)^2}{(\omega_l-\omega)^2 + (\Gamma/2)^2}, \tag{36}$$

$$\mathcal{A} = \frac{\gamma\gamma_r/2}{(\omega_l-\omega)^2 + (\Gamma/2)^2}, \tag{37}$$

obtained earlier in Refs. [14,15,16,17,18]. Eq. (37) contains the very important result: Light absorption in a QW is absent completely when $\gamma = 0$. [17,18]

Let us consider two limit cases: $\gamma_r << \gamma$ and $\gamma_r >> \gamma$. In the case $\gamma_r << \gamma$ the perturbation theory on light-electron interaction is applicable in the lowest order. In this approximation, the values of the light absorption and reflection coefficients are of the second and fourth order on interaction, respectively. From Eq. (36) and Eq. (37) we obtain

$$\mathcal{R} \simeq \frac{\pi\hbar}{2}\frac{\gamma_r^2}{\gamma}\Delta_\gamma[\hbar(\omega_l-\omega)], \tag{38}$$

$$\mathcal{A} \simeq \pi\hbar\gamma_r\Delta_\gamma[\hbar(\omega_l-\omega)], \tag{39}$$

where $\Delta_\gamma(E)$ is determined in Eq. (2).

At $\gamma_r << \gamma$

$$\mathcal{A} << 1, \quad \mathcal{R} << \mathcal{A}. \tag{40}$$

In the opposite case $\gamma_r >> \gamma$ in Eq. (36) and Eq. (37) Γ has to be substituted approximately by γ_r. Then we obtain that in the resonance $\mathcal{R}(\omega_l = \omega) = 1$, which means the total light reflection, $\mathcal{A} << 1$.

Thus, light absorption is small for both cases $\gamma_r << \gamma$ and $\gamma_r >> \gamma$. The maximum value $\mathcal{A}(\omega_l = \omega) = 1/2$ is reached in the case $\gamma_r = \gamma$.

With the help of Eq. (33), Eq. (34) and Eq. (21), Eq. (22) we obtain for the three-level system

$$\mathcal{R} = \frac{1}{4Z}\{[\gamma_{r1}(\omega_l-\omega_2) + \gamma_{r2}(\omega_l-\omega_1)]^2 + \\ + (\gamma_{r1}\gamma_2 + \gamma_{r2}\gamma_1)^2/4\}, \tag{41}$$

$$\mathcal{A} = \frac{1}{2Z}\{\gamma_{r1}\gamma_1[(\omega_l-\omega_2)^2 + (\gamma_2/2)^2] + \\ + \gamma_{r2}\gamma_2[(\omega_l-\omega_1)^2 + (\gamma_1/2)^2]\}, \tag{42}$$

where

$$Z = [(\omega_l-\Omega_1)^2 + (G_1/2)^2][(\omega_l-\Omega_2)^2 + (G_2/2)^2]. \tag{43}$$

It follows from Eq. (42) that light absorption by the three-level system is equal 0, if

$$\gamma_1 = \gamma_2 = 0. \tag{44}$$

Applying the fact that the quantities $(\Omega - iG/2)_{1,2}$ are the roots of Eq. (23), we transform the denominator in Eq. (43) to the form

$$Z = [(\omega_l-\omega_1)(\omega_l-\omega_2) - (\gamma_{r1}\gamma_2 + \gamma_{r2}\gamma_1 + \gamma_1\gamma_2)/4]^2 + \\ + [(\omega_l-\omega_1)\Gamma_2 + (\omega_l-\omega_2)\Gamma_1]^2/4. \tag{45}$$

Let us obtain the simplified expressions for $\mathcal{R}$ and $\mathcal{A}$ for the different limits. It is convenient to use Eq. (41) and Eq. (42) with the substitution Eq. (43) or, sometimes, with Eq. (45). The first limit case corresponds to the inequalities

$$\gamma_{r1(2)} << \gamma_{1(2)}, \tag{46}$$

when the perturbation theory on light-electron interaction is applicable what corresponds to neglecting by the term $4\pi\chi(\omega)$ in the RHS of Eq. (14). Under condition Eq. (46) in the RHS of Eq. (43) we suppose $\Omega_{1(2)} \simeq \omega_{1(2)}, G_{1(2)} = \gamma_{1(2)}$ and obtain

$$\mathcal{R} \simeq \frac{(\gamma_{r1}/2)^2}{(\omega_l-\omega_1)^2 + (\gamma_1/2)^2} + \frac{(\gamma_{r2}/2)^2}{(\omega_l-\omega_2)^2 + (\gamma_2/2)^2} + \\ + \frac{\gamma_{r1}\gamma_{r2}}{2} \times \\ \times \frac{(\omega_l-\omega_1)(\omega_l-\omega_2) + \gamma_1\gamma_2/4}{[(\omega_l-\omega_1)^2 + (\gamma_1/2)^2][(\omega_l-\omega_2)^2 + (\gamma_2/2)^2]}, \tag{47}$$

$$\mathcal{A} \simeq \frac{\gamma_{r1}\gamma_1/2}{(\omega_l-\omega_1)^2+(\gamma_1/2)^2} + \frac{\gamma_{r2}\gamma_2/2}{(\omega_l-\omega_2)^2+(\gamma_2/2)^2}. \quad (48)$$

According to Eq. (48) the absorption coefficient $\mathcal{A}$ is the sum of contributions Eq. (39) from the levels 1 and 2, because under condition Eq. (46) absorption is linear on the constants γ_{r1} and γ_{r2}. According to Eq. (47) the reflection coefficient $\mathcal{R}$ is square-law on γ_{r1} and γ_{r2}. Therefore it contains an interferential contribution besides the separate levels contributions. The functions $\mathcal{A}(\omega_l)$ and $\mathcal{R}(\omega_l)$ are represented on Fig. 2 under condition Eq. (46) for the case $\gamma_{r1}=\gamma_{r2}$, $\gamma_1=\gamma_2$.

Generalizing Eq. (48) for the case of the arbitrary number of the energy levels and under condition Eq. (28) one obtains

$$\mathcal{A}^{many} \simeq \frac{1}{2}\sum_{\rho}\frac{\gamma_{r\rho}\gamma_\rho}{(\omega_l-\omega_\rho)^2+(\gamma_\rho/2)^2}. \quad (49)$$

The next limit case (opposite to the preceding one) is determined by the conditions

$$\gamma_{r1(2)} >> \gamma_{1(2)}. \quad (50)$$

We suppose $\gamma_1=\gamma_2=0$ in the RHS of Eq. (41) and Eq. (42). Then $\mathcal{A}=0$ and one obtains

$$\mathcal{R}\,(\gamma_1=\gamma_2=0) = [(\gamma_{r1}+\gamma_{r2})/2]^2(\omega_l-\omega_0)^2 \times \\ \times [(\omega_l-\omega_1)^2(\omega_l-\omega_2)^2 + \\ + [(\gamma_{r1}+\gamma_{r2})/2]^2(\omega_l-\omega_0)^2]^{-1}, \quad (51)$$

where

$$\omega_0 = \frac{\omega_1\gamma_{r2}+\omega_2\gamma_{r1}}{\gamma_{r1}+\gamma_{r2}}. \quad (52)$$

The peculiar properties of light reflection from the three-level system follow from Eq. (51) in the case of the dominant inverse radiative lifetimes. *At any quantities γ_{r1} and γ_{r2} in the point $\omega_l=\omega_0$ light reflection equals 0; in the points $\omega_l=\omega_1$ and $\omega_l=\omega_2$ the reflection coefficient equals 1.* Let us suppose $\gamma_{r1}=\gamma_{r2}=\gamma_r$. Then $\mathcal{R}=0$ at $\omega_l=(\omega_1+\omega_2)/2$. At $\gamma_r >> (\omega_1-\omega_2)$ in the point $\omega_l=(\omega_1+\omega_2)/2$ there is a narrow minimum descending to $\mathcal{R}=0$. The half-width of this minimum on the half-depth equals to $(\omega_1-\omega_2)^2/2^{3/2}\gamma_r$.

The functions $\mathcal{R}(\omega_l)$ are represented in Fig. 3b under conditions

$$\gamma_1=\gamma_2=\gamma, \qquad \gamma_{r1}=\gamma_{r2}=\gamma_r, \quad \gamma << \gamma_r. \quad (53)$$

The functions $\mathcal{A}(\omega_l)$ are represented on Fig. 3a under the same condition Eq. (53). Note a peculiar turn of $\mathcal{A}(\omega_l)$ curves at a fixed quantity γ and at growing γ_r. Transiting from the curve *1* to the curve *6* the quantity $\hbar\gamma_r$ takes a successive row of values: 0.002, 0.005, 0.008, 0.04, 0.125, 0.5. There are two maxima on the first curve at $\hbar\gamma_r=0.002$. On the consecutive curves there is one maximum which reaches its largest value $\mathcal{A}_{max}=0.5$ on the curve *5* ($\hbar\gamma_r=0.125$), and afterwards its height $\mathcal{A}_{max}$ diminishes. The quantity $\mathcal{A}_0$ corresponding to the central point $\omega_l=(\omega_1+\omega_2)/2$ and coinciding with the height $\mathcal{A}_{max}$ on the curves *2* - *6* is described by the precise formula

$$\mathcal{A}_0 = \frac{4\gamma_r\gamma[(\omega_1-\omega_2)^2+\gamma^2]}{[(\omega_1-\omega_2)^2+2\gamma_r\gamma+\gamma^2]^2}, \quad (54)$$

which follows from Eq. (42) at $\omega_l=(\omega_1+\omega_2)/2$. The quantity $\mathcal{A}_0$ maximizes at

$$\gamma_{r0} = \frac{(\omega_1-\omega_2)^2+\gamma^2}{2\gamma}. \quad (55)$$

Substituting the values $\hbar(\omega_1-\omega_2)=0.005$ and $\hbar\gamma=0.0001$ in Eq. (55), what corresponds Fig. 3a, one obtains $\hbar\gamma_{r0}=0.125$ and $\mathcal{A}_{max}=\mathcal{A}_0=0.5$ (the curve *5* on Fig.3a).

The next limit realizes when the reverse lifetimes $\gamma_{1(2)}$ and $\gamma_{r1(2)}$ are small in comparison with the interlevel separation $\omega_1-\omega_2$. Let us suppose

$$\gamma_{1(2)} << \omega_1-\omega_2, \qquad \gamma_{r1(2)} << \omega_1-\omega_2. \quad (56)$$

The interrelation between the non-radiative $\gamma_{1(2)}$ and radiative $\gamma_{r1(2)}$ lifetimes can be an arbitrary one. Then, if the frequency ω_l is close to the resonance with one of the levels , let us say $\omega_l \simeq \omega_1$, one obtains from Eq. (41) and Eq. (42)

$$\mathcal{R} \simeq \frac{\gamma_{r1}^2/4}{(\omega_l-\omega_1)^2+(\Gamma_1/2)^2}, \\ \mathcal{A} \simeq \frac{\gamma_{r1}\gamma_1/2}{(\omega_l-\omega_1)^2+(\Gamma_1/2)^2}, \quad (57)$$

which coincides with the results of Eq. (36) and Eq. (37) for the two-level system.

Lastly we consider the case of the merging curves. Supposing

$$\gamma_1=\gamma_2=\gamma, \quad \omega_1=\omega_2=\omega, \quad \gamma_{r1}=\gamma_{r2}=\gamma_r, \quad (58)$$

we obtain from Eq. (41) and Eq. (42)

$$\mathcal{R} \simeq \frac{\gamma_r^2}{(\omega_l-\omega)^2+(\gamma_r+\gamma/2)^2}, \quad (59)$$

$$\mathcal{A} \simeq \frac{\gamma_r\gamma}{(\omega_l-\omega)^2+(\gamma_r+\gamma/2)^2}. \quad (60)$$

These results are similar to Eq. (36) and Eq. (37) for the two-level system, but *the quantity γ_r is reduplicated*. That means that *in the case of a double degenerated excited level the formulae for the two-level system with the reduplicated value γ_r are applicable.*

V. EHP RADIATIVE LIFETIME FOR QW IN MAGNETIC FIELD.

One could calculate the EHP inverse radiative lifetime in magnetic field indirectly: calculating the absorption coefficient $\mathcal{A}$ with the help of the perturbation theory and then, comparing the obtained result with Eq. (39), to calculate γ_r. The second way is the direct calculation of $\chi(\omega)$ (see Eq. (15)). But we shall demonstrate the third way: The direct calculation of γ_r with the help of the Fermi Golden Rule

$$\gamma_{r\eta} = \frac{2\pi}{\hbar}\sum_s |\langle s|U|\eta\rangle|^2 \delta(\hbar\omega_s - E_\eta). \tag{61}$$

We ascertained that all three calculation methods led to the same results for γ_r.

In Eq. (61) $|\eta\rangle, |s\rangle$ are the secondary quantized wave functions. The initial state wave function $|\eta\rangle$ describes the EHP with the index set η; the final state $|s\rangle$ describes the photon with the index set s. The interaction U is written as

$$U = -\frac{1}{c}\int d\mathbf{r}\mathbf{A}(\mathbf{r})\mathbf{j}(\mathbf{r}), \tag{62}$$

$$\mathbf{A}(\mathbf{r}) = \left(\frac{2\pi\hbar}{V_0}\right)^{1/2}\frac{c}{n}\sum_s \omega_s^{-1/2}(c_s\mathbf{e}_s e^{i\kappa\mathbf{r}} + c_s^+\mathbf{e}_s^* e^{-i\kappa r}). \tag{63}$$

The index set s involves the wave vector κ and the polarization index i (accepting two values), $c_s^+(c_s)$ is the photon creation (annihilation) operator , $\mathbf{e}_s$ and $\hbar\omega_s$ are the polarization vector and energy, respectively; V_0 is the normalization volume. The charge current density operator is given by

$$\mathbf{j}(\mathbf{r}) = \frac{e}{m_0}\sum_\xi [\mathbf{p}_{cv}F_\xi^*(\mathbf{r})a_\xi^+ + \mathbf{p}_{cv}^* F_\xi(\mathbf{r})a_\xi], \tag{64}$$

where m_0 is the bare electron mass, $a_\xi^+(a_\xi)$ is the EHP creation (annihilation) operator, ξ is the index set,

$$F_\xi(\mathbf{r}) = \Psi_\xi(\mathbf{r}_e = \mathbf{r}_h = \mathbf{r}), \tag{65}$$

$\Psi_\xi(\mathbf{r}_e, \mathbf{r}_h)$ is the smooth part of the EHP wave function (in the effective mass approximation), depending on the radius vectors $\mathbf{r}_e, \mathbf{r}_h$ of the electron and hole, respectively, $\mathbf{p}_{cv}$ is the interband matrix element of the momentum operator. Let us stress that the index set ξ includes the indexes c and v of the conductivity and valence bands, which can be degenerate ones.

Let us determine the wave functions $\Psi_\xi(\mathbf{r}_e, \mathbf{r}_h)$. That is well known that the electron (hole) wave functions, corresponding to the gauge $\mathbf{A} = \mathbf{A}(0, xH, 0)$ of the vector-potential, are

$$\psi^{e(h)}_{c(v),n,k_y,l}(\mathbf{r}) = \phi_n(x \pm a_H^2 k_y)\frac{1}{\sqrt{L_y}}e^{ik_y y}\varphi_{c(v)l}(z). \tag{66}$$

The upper sign "+" corresponds to the electron, $a_H = \sqrt{c\hbar/|e|H}$, L_y is the normalization length,

$$\phi_n(x) = \frac{1}{\sqrt{a_H}}f_n(x/a_H),$$
$$f_n(t) = \frac{1}{\sqrt{\pi^{1/2}2^n n!}}e^{-t^2/2}H_n(t), \tag{67}$$

$H_n(t)$ is the Hermitian polynomial. The real functions $\varphi_{c(v)l}(z)$, corresponding to the size-quantization quantum number l for the QW of the finite depth, can be found, for instance, in Ref. [26].

At first we introduce an EHP wave function set consisting of a product of the wave functions Eq. (66) for electrons and holes and characterized by the index set

$$\zeta \to c, v, n_e, n_h, k_{ey}, k_{hy}, l_e, l_h. \tag{68}$$

$$\Psi_\zeta(\mathbf{r}_e, \mathbf{r}_h) = \psi^e_{c,n_e,k_{ey},l_e}(\mathbf{r}_e)\psi^h_{v,n_h,k_{hy},l_h}(\mathbf{r}_h). \tag{69}$$

However one cannot calculate the EHP radiative lifetime with the help of Eq. (69). One has to introduce the wave functions for a state of the EHP created by light from the ground state of the system in a QW. Therefore we go to the wave functions

$$\Psi_\xi(\mathbf{r}_e, \mathbf{r}_h) = \sqrt{\frac{2\pi a_H^2}{L_x L_y}}\sum_{k_{ey},k_{hy}} \delta_{K_y,k_{ey}+k_{hy}} e^{-ia_H^2 K_x k_y} \times$$
$$\times \Psi_\zeta(\mathbf{r}_e, \mathbf{r}_h), \tag{70}$$

where

$$k_y = \frac{k_{ey}m_h - k_{hy}m_e}{M}, \qquad M = m_e + m_h,$$

$m_e(m_h)$ is the electron (hole) effective mass.

The functions of type of Eq. (70) have been introduced in Ref. [27], see also Refs. [28,29,30]. The index set is

$$\xi \to c, v, n_e, n_h, \mathbf{K}_\perp, l_e, l_h. \tag{71}$$

It has been shown in Ref. [29] that the functions Eq. (70) are the eigenfunctions of the EHP momentum operator $\hat{\mathbf{P}}_\perp$ in the xy plane, corresponding to the eigenvalues $\hbar\mathbf{K}_\perp$. The operator $\hat{\mathbf{P}}_\perp$ is determined in Ref. [31].

With the help of the coordinates

$$\mathbf{r}_\perp = \mathbf{r}_{e_\perp} - \mathbf{r}_{h\perp}, \qquad \mathbf{R}_\perp = \frac{m_h\mathbf{r}_{h\perp} + m_e\mathbf{r}_{e\perp}}{M}$$

of the relative and whole movement of the electron and hole in the xy plane and summarizing on k_{ey}, k_{hy} in the RHS of Eq. (70), one obtains [28,29,30]

$$\Psi_{\xi}(\mathbf{r}_e,\mathbf{r}_h)=\frac{1}{\sqrt{2\pi a_H^2 L_x L_y}}\times$$
$$\times\exp\left[i(K_xX+K_yY)-i\frac{yX}{a_H^2}\right]\times$$
$$\times\exp\left[i\frac{m_e-m_h}{2M}(-\mathbf{K}_\perp\mathbf{r}_\perp+a_H^2K_xK_y)+\frac{xy}{a_H^2}\right]\times$$
$$\times\mathcal{K}_{n_e,n_h}\left(\frac{\mathbf{r}_\perp-\mathbf{r}_{\perp 0}}{a_H}\right)\varphi_{cl_e}(z_e)\varphi_{vl_h}(z_h), \qquad (72)$$

where

$$\mathbf{r}_{\perp 0}=a_H^2\frac{\mathbf{H}\times\mathbf{K}_\perp}{H}.$$

The function $\mathcal{K}_{n,m}(\mathbf{p})$ is determined by [28]

$$\mathcal{K}_{n,m}(\mathbf{p})=\left[\frac{min(n!,m!)}{max(n!,m!)}\right]^{1/2}i^{|n-m|}e^{-p^2/4}\times$$
$$\times\left(\frac{p}{\sqrt{2}}\right)^{|n-m|}\exp[i(\phi-\pi/2)(n-m)]\times$$
$$\times L_{min(n,m)}^{|n-m|}(p^2/2), \qquad (73)$$

where

$$p=\sqrt{p_x^2+p_y^2},\qquad \phi=\arctan(p_y/p_x),$$

$L_m^n(x)$ is the Laguerre polynomial. The energy eigenvalues of the states Eq. (72) are

$$E_\xi=\hbar\omega_\xi=E_g+\varepsilon_{le}^e+\varepsilon_{lh}^h+$$
$$+\hbar\Omega_e(n_e+1/2)+\hbar\Omega_h(n_h+1/2), \qquad (74)$$

where E_g is the energy gap.

With the help of Eq. (72) one obtains

$$F_\xi(\mathbf{r})=\frac{1}{\sqrt{2\pi a_H^2L_xL_y}}\exp\left(i\frac{m_e-m_h}{2M}a_H^2K_xK_y\right)\times$$
$$\times\mathcal{K}_{n_e,n_h}\left(-\frac{\mathbf{r}_{\perp 0}}{a_H}\right)e^{i\mathbf{K}_\perp\mathbf{r}_\perp}\varphi_{cl_e}(z)\varphi_{vl_h}(z). \qquad (75)$$

Let us calculate $\gamma_{r\xi}$ for the index set ξ. With the help of Eqs. (61) - (64) one obtains

$$\gamma_{r\xi}=\frac{(2\pi e)^2}{V_0(nm_0)^2}\sum_s\frac{|\mathbf{e}_s\mathbf{p}_{cv}|^2}{\omega_s}\left|\int d\mathbf{r}e^{-i\kappa\mathbf{r}}F_\xi(\mathbf{r})\right|^2\times$$
$$\times\delta(\hbar\omega_s-E_\xi). \qquad (76)$$

The substitution of Eq. (75) in Eq. (76) results in

$$\gamma_{r\xi}=\left(\frac{e}{a_Hnm_0}\right)^2\frac{1}{\hbar\omega_\xi}B_{n_e,n_h}(K_\perp)\frac{2\pi}{L_z}\sum_{i,\kappa}\delta_{\kappa_\perp,\mathbf{K}_\perp}\times$$
$$\times|\mathbf{e}_s\mathbf{p}_{cv}|^2|R_{l_e,l_h}(\kappa_z)|^2\delta(\omega_s-\omega_\xi), \qquad (77)$$

where

$$B_{n,m}(K_\perp)=\left|\mathcal{K}_{n,m}\left(-\frac{\mathbf{r}_{\perp 0}}{a_H}\right)\right|^2=\frac{min(n!,m!)}{max(n!,m!)}\times$$
$$\times\exp\left(-\frac{a_H^2K_\perp^2}{2}\right)\left(\frac{a_H^2K_\perp^2}{2}\right)^{|n-m|}\times$$
$$\times\left[L_{min(n,m)}^{|n-m|}\left(\frac{a_H^2K_\perp^2}{2}\right)\right]^2, \qquad (78)$$

$$R_{l_e,l_h}(k)=\int_{-\infty}^{\infty}dze^{-ikz}\varphi_{cl_e}(z)\varphi_{vl_h}(z). \qquad (79)$$

Summing up in the RHS of Eq. (77) on $\kappa_\perp$ and taking into account that $\omega_s=c\kappa/n$, $\kappa=\sqrt{\kappa_\perp^2+\kappa_z^2}$, one obtains

$$\gamma_{r\xi}=\frac{2\pi e^2}{L_zm_0^2\hbar\omega_\xi a_H^2cn}\times$$
$$\times B_{n_e,n_h}(K_\perp)\left|R_{l_e,l_h}(\sqrt{(\omega_\xi n/c)^2-K_\perp^2})\right|^2\times$$
$$\times\sum_{i,\kappa_z}|\mathbf{e}_s\mathbf{p}_{cv}|^2\delta\left(\sqrt{K_\perp^2+\kappa_z^2}-\omega_\xi n/c\right). \qquad (80)$$

Summing up on κ_z at $K_\perp<\omega_\xi n/c$, one obtains

$$\gamma_{r\xi}=\frac{e^2\Omega_0}{\hbar^2c^2m_0}B_{n_e,n_h}(K_\perp)\times$$
$$\times\left|R_{l_e,l_h}(\sqrt{(\omega_\xi n/c)^2-K_\perp^2})\right|^2\frac{1}{\sqrt{(\omega_\xi n/c)^2-K_\perp^2}}\times$$
$$\times\sum_i[|\mathbf{e}_i(\kappa^+)\mathbf{p}_{cv}|^2+|\mathbf{e}_i(\kappa^-)\mathbf{p}_{cv}|^2];$$
$$\gamma_{r\xi}=0,\qquad K_\perp>\omega_\xi n/c, \qquad (81)$$

where $\Omega_0=|e|H/m_0c$, $\kappa^+(\kappa^-)$ is the photon wave vector with the component $\mathbf{K}_\perp$ in the xy plane and with the z-component equal to $\pm\sqrt{(\omega_\xi n/c)^2-K_\perp^2}$. Thus the EHP with the energy $\hbar\omega_\xi$ and the wave vector $\mathbf{K}_\perp$ in the xy plane emits the photons with the energies $\hbar\omega_\xi$ and wave vectors κ^+ and κ^- if $K_\perp<\omega_\xi n/c$.

Let us direct the x axis along $\mathbf{K}_\perp$. Summarizing on the photon polarizations at $K_\perp<\omega_\xi n/c$ one obtains

$$\gamma_{r\xi}=\frac{2e^2\Omega_0}{\hbar^2cnm_0\omega_\xi}B_{n_e,n_h}(K_\perp)\times$$
$$\times\left|R_{l_e,l_h}(\sqrt{(\omega_\xi n/c)^2-K_\perp^2})\right|^2\times$$
$$\times\left[\frac{\sqrt{(\omega_\xi n/c)^2-K_\perp^2}}{\omega_\xi n/c}|p_{cvx}|^2+\right.$$
$$+\frac{\omega_\xi n/c}{\sqrt{(\omega_\xi n/c)^2-K_\perp^2}}|p_{cvy}|^2+$$
$$\left.+\frac{K_\perp^2|p_{cvz}|^2}{(\omega_\xi n/c)\sqrt{(\omega_\xi n/c)^2-K_\perp^2}}\right]. \qquad (82)$$

Eq. (82) can be compared with the result from Ref. [14] regarding to the exciton energy level in a QW at $H = 0$. It turns out that the dependencies from the components p_{cvx}, p_{cvy} and p_{cvz} coincide and the result $\gamma_{r\xi} = 0$ at $K_\perp > \omega_\xi n/c$ coincides also. The RHS of Eq. (82) contains the factors Ω_0 and $B_{n_e,n_h}(K_\perp)$ due to magnetic field. The factor $\left|R_{l_e,l_h}(\sqrt{(\omega_\xi n/c)^2 - K_\perp^2})\right|^2$ is absent in Ref. [14] because the QWs with $d << c/n\omega_l$ have been considered. We did not apply such a restriction obtaining Eq. (82).

For the narrow QWs with $d << \lambda$ one has

$$R_{l_e,l_h}\left(\sqrt{(\omega_\xi n/c)^2 - K_\perp^2}\right) \simeq$$
$$\simeq \pi_{l_e,l_h} = \int_{-\infty}^{\infty} dz \varphi_{cle}(z)\varphi_{vlh}(z), \qquad (83)$$

and for infinitely deep QWs

$$\pi_{l_e,l_h} = \delta_{l_e,l_h}. \qquad (84)$$

For QWs with $d \simeq c/n\omega_l$ the quantity $\left|R_{l_e,l_h}(\sqrt{(\omega_\xi n/c)^2 - K_\perp^2})\right|^2$ depends on d and diminishes with growth d, therefore the quantity $\gamma_{r\xi}$ diminishes also. In the limit $d >> \lambda$ the quantity $\gamma_{r\xi} \to 0$ as in a bulk crystal. [12]

We are interested in obtaining the quantities Eq. (82) at $K_\perp = 0$ because we calculate the absorption and reflection coefficients for the light normal incidence. Because $B_{n_e,n_h}(0) = \delta_{n_e,n_h}$, let us calculate $\gamma_{r\xi_0}$ for the index set

$$\xi_0 \to c, v, n_e = n_h, \mathbf{K}_\perp = 0, l_e, l_h. \qquad (85)$$

With the help of Eq. (82) one obtains

$$\gamma_{r\xi_0} = \frac{2e^2\Omega_0}{\hbar cn} |R_{l_e,l_h}(\omega_{\xi_0} n/c)|^2 \frac{|p_{cvx}|^2 + |p_{cvy}|^2}{m_0\hbar\omega_{\xi_0}}, \qquad (86)$$

where

$$\hbar\omega_{\xi_0} = E_g + \varepsilon_{le}^c + \varepsilon_{lh}^h + \hbar\Omega_\mu(n_e + 1/2), \qquad (87)$$

$$\Omega_\mu = \frac{|e|H}{\mu c}, \qquad \mu = \frac{m_e m_h}{M}.$$

It follows from Eq. (86) that the EHP inverse radiative lifetime *is proportional to magnetic field H* if the energy $\hbar\omega_{\xi_0}$ weakly depends on H (approximately $E_{\xi_0} \simeq \tilde{E}_g, \quad \tilde{E}_g = E_g + \varepsilon_{le}^e + \varepsilon_{lh}^h$).

Let us calculate $\gamma_{r\xi_0}$ for the band model of $GaAs$. The conductivity band is twice degenerated (on the spin) and the index c takes two values : $c = 1$ or $c = 2$. The valence band (for heavy holes) is twice degenerate also: $v = 1$ or $v = 2$. Two EHP sorts are possible: First, with the indexes $c = 1, v = 1$, second, with indexes $c = 2, v = 2$. These EHP sorts I and II differ by values $\mathbf{p}_{cv}$, which are

$$\mathbf{p}_{cv}^I = \frac{1}{\sqrt{2}} p_{cv}(\mathbf{e}_x - i\mathbf{e}_y), \quad \mathbf{p}_{cv}^{II} = \frac{1}{\sqrt{2}} p_{cv}(\mathbf{e}_x + i\mathbf{e}_y). \qquad (88)$$

For the proposed model the wave functions $\varphi_{cle}(z)$ and $\varphi_{vlh}(z)$ do not depend on indexes c and v. When the circular polarizations are used, every polarization (left or right relatively of the axis z) is toughly linked with the EHP sort (I ore II) because the EHP – light interaction is proportional to $\mathbf{e}_i\mathbf{p}_{cv}$.

For the model Eq. (88) one obtains from Eq. (86) the result for any sort [‡]

$$\gamma_{r\xi_0} = \frac{2e^2\Omega_0}{\hbar cn} |R_{l_e,l_h}(\omega_\xi n/c)|^2 \frac{p_{cv}^2}{m_0\hbar\omega_{\xi_0}}. \qquad (89)$$

Applying the approximation (83) for the narrow QWs with $d << \lambda$, we obtain

$$\gamma_{r\xi_0} \simeq \frac{2e^2\Omega_0}{\hbar cn} \pi_{l_e,l_h}^2 . \frac{p_{cv}^2}{m_0\hbar\omega_{\xi_0}}. \qquad (90)$$

Applying the parameters for GaAs from Ref. [**?**] and the approximation of Eq. (84) and supposing $\hbar\omega_{\xi_0} \simeq E_g$, we obtain the numerical estimation

$$\hbar\gamma_r \simeq 5.35 \cdot 10^{-5} \frac{H}{H_{res}} eV, \qquad (91)$$

where H_{res} appropriates to the magnetopolaron A (Fig. 1), i. e. equals to $H_{res} = m_e c\omega_{LO}/|e|$.

VI. INVERSE RADIATIVE LIFETIMES OF MAGNETOPOLARON.

Let us consider the polaron A (Fig. 1) in a combination with the hole characterized by the size-quantization quantum number l_h and the Landau quantum number $n_h = 1$. Close to the resonance at $\omega_{LO} = \Omega_e$ the EHP energy splits into two magnetopolaron branches with the inverse radiative lifetimes γ_{ra} and γ_{rb}, according to designations of Ref. [26]. The index $a(b)$ corresponds to the upper (lower) magnetopolaron term.

To calculate γ_{ra} and γ_{rb} we will use Eq. (76), determining the function $F_\xi(\mathbf{r})$ from the RHS of Eq. (76). At first we consider the EHP wave functions under conditions of the magnetophonon resonance [§]

$$\Psi_\Pi(\mathbf{r}_e, \mathbf{r}_h)|0\rangle = \psi_{1,k_{hy},l_h}^h(\mathbf{r}_h)\Theta_{p,k_{ey},l_e}(\mathbf{r}_e)|0\rangle. \qquad (92)$$

The hole wave function $\psi_{1,k_{hy},l}^h(\mathbf{r})$ is defined in Eq. (66). The polaron wave function $\Theta_{p,k_y,l}(\mathbf{r}_e)|0\rangle$ is calculated in Ref. [26]. The index set in Eq. (92) is

$$\Pi \to c, v, p; k_{ey}, k_{hy}, l_e, l_h.$$

[‡]The corresponding formula from Ref. [25] coincides with Eq. (89) at $n = 1$.

[§]Here and below the indexes c and v are omitted.

The index p takes two values : a or b. The function $|0\rangle$ appropriates to the phonon vacuum . The operator $\Theta_{p,k_y,l}(\mathbf{r}_e)$, according to Ref. [26], can be written as

$$\Theta_{p,k_y,l}(\mathbf{r}) = \Theta^0_{p,k_y,l}(\mathbf{r}) + \Theta^1_{p,k_y,l}(\mathbf{r}), \tag{93}$$

where

$$\Theta^0_{p,k_y,l}(\mathbf{r}) = Q^{1/2}_{0p}\psi^e_{1,k_y,l}(\mathbf{r}), \tag{94}$$

$$\Theta^1_{p,k_y,l}(\mathbf{r}) = \frac{Q^{1/2}_{1p}}{|A|}\sum_\nu e^{ia_H q_x(k_y-q_y/2)}U^*(\nu)\psi^e_{0,k_y-q_y}(\mathbf{r})b^+_\nu. \tag{95}$$

In the RHS of Eq. (94) and Eq. (95) $\psi^e_{n,k_y,l}(\mathbf{r})$ are the electron wave functions determined in Eq. (66); ν is the phonon index set, including the transverse component $\mathbf{q}_\perp$; b^+_ν is the phonon creation operator; $U(\nu)$ is the quantity proportional to electron-phonon interaction and determined in Ref. [26]; $A^2 = \sum_\nu |U(\nu)|^2$. The polaron wave function $\Theta_{p,k_y,l}(\mathbf{r})|0\rangle$ and the corresponding energy eigenvalue (measured from the energy level ε^e_{le})

$$E_p = \hbar\Omega_e + \frac{1}{2}\hbar\omega_{LO} \pm \sqrt{(\lambda/2)^2 + A^2}, \qquad \lambda = \hbar(\Omega_e - \omega_{LO}) \tag{96}$$

have been calculated in Ref. [26], where it was assumed that all the LO phonons, composing the polaron, had the same non-dispersal frequency ω_{LO}. The coefficients in Eq. (94) and Eq. (95) are

$$Q_{0p} = \frac{1}{2}\left(1 \pm \frac{\lambda}{\sqrt{\lambda^2+4A^2}}\right), \quad Q_{1p} = \frac{1}{2}\left(1 \mp \frac{\lambda}{\sqrt{\lambda^2+4A^2}}\right), \tag{97}$$

where the upper (lower) sign refers to the term $p = a$ ($p = b$).

It follows from Eq. (93) that the polaron wave function is represented by the linear combination of two functions of the electron-phonon system, one of which corresponds to the electron with the Landau quantum number $n = 1$ and to the phonon vacuum, and another corresponds to the electron with $n = 0$ and the LO phonon. The coefficients Q_{0p} and Q_{1p} are the probabilities of finding the system in these states designated with indexes 0 and 1, respectively.

In the resonance $\Omega_e = \omega_{LO}$ the coefficients $Q^{res}_{0p} = Q^{res}_{1p} = 1/2$ and the energy

$$E^{res}_p = \frac{3}{2}\hbar\omega_{LO} \pm \sqrt{A^2}, \quad \Delta E = E^{res}_a - E^{res}_b = 2\sqrt{A^2}. \tag{98}$$

To calculate the inverse radiative lifetimes γ_{ra} and γ_{rb} it is necessary to introduce the eigenfunctions of the EHP momentum operator $\hat{\mathbf{P}}$. Analogically to Eq. (70) we introduce the functions

$$\Psi_\eta(\mathbf{r}_e,\mathbf{r}_h)|0\rangle = \sqrt{\frac{2\pi a_H^2}{L_xL_y}} \times \times \sum_{k_{ey},k_{hy}} \delta_{K_y,k_{ey}+k_{hy}} e^{-ia_H^2K_xk_y}\Psi_\Pi(\mathbf{r}_e,\mathbf{r}_h)|0\rangle, \tag{99}$$

characterized by the index set

$$\eta \to c, v, p; \mathbf{K}_\perp, l_e, l_h. \tag{100}$$

Determining $\gamma_{r\eta}$ according to Eq. (76), we substitute

$$F_\eta(\mathbf{r}) = \langle 0|\Psi_\eta(\mathbf{r}_e = \mathbf{r}_h = \mathbf{r})|0\rangle, \tag{101}$$

instead of $F_\xi(\mathbf{r})$ and the energy

$$E_\eta = \hbar\omega_\eta = E_g + \varepsilon^e_{le} + \varepsilon^h_{lh} + \frac{3}{2}\hbar\Omega_h + E_p. \tag{102}$$

instead of the energy E_ξ. So long as $\langle 0|b^+_\nu|0\rangle = 0$ only the first term from the RHS of Eq. (93) contributes into the RHS of Eq. (101). And for $F_\eta(\mathbf{r})$ we obtain the expression distinguishing from Eq. (75) at $n_e = n_h = 1$ only by the additional factor $Q^{1/2}_{0p}$. If the x axis is directed along $\mathbf{K}_\perp$, the result for the inverse radiative lifetimes $\gamma_{r\eta}$ is analogical to Eq. (82)

$$\begin{aligned}\gamma_{r\eta} = {} & \frac{2Q_{0p}e^2\Omega_0}{\hbar^2 cnm_0\omega_\eta}B_{11}(K_\perp) \times \\ & \times \left|R_{l_e,l_h}\left(\sqrt{(\omega_\eta n/c)^2 - K_\perp^2}\right)\right|^2 \times \\ & \times \left[\frac{\sqrt{(\omega_\eta n/c)^2 - K_\perp^2}}{\omega_\eta n/c}|p_{cvx}|^2 + \right. \\ & + \frac{\omega_\eta n/c}{\sqrt{(\omega_\eta n/c)^2 - K_\perp^2}}|p_{cvy}|^2 + \\ & \left. + \frac{K_\perp^2}{(\omega_\eta n/c)\sqrt{(\omega_\eta n/c)^2 - K_\perp^2}}|p_{cvz}|^2\right]; \ K_\perp < \omega_\eta n/c; \\ \gamma_{r\eta} = {} & 0; \quad K_\perp > \omega_\eta n/c. \end{aligned} \tag{103}$$

For the case $\mathbf{K}_\perp = 0$ we introduce the index set

$$\eta_0 \to c, v, p; \mathbf{K}_\perp = 0, l_e, l_h. \tag{104}$$

Then for the quantities $\gamma_{r\eta_0}$ we obtain the formulae distinguishing from Eq. (86), Eq. (89) and Eq. (90) only by the substitution the indexes η instead of ξ_0 (as long as $\omega_{\eta_0} = \omega_\eta$) and by the factor Q_{0p}.

Instead of Eq. (90) we obtain

$$\gamma_{r\eta_0} \simeq \frac{2Q_{0p}e^2}{\hbar c}\frac{\Omega_0 p_{cv}^2}{nm_0\hbar\omega_\eta}\pi^2_{l_e,l_h}. \tag{105}$$

The substitution ω_η instead of ω_{ξ_0} is not essential as long as approximately $\omega_{\xi_0} \simeq \omega_\eta \simeq \tilde{E}_g/\hbar$. The tough dependence of the inverse radiative lifetime from the index p and from the value λ (characterizing the deviation from the resonance) is determined by the factor Q_{0p}. In the resonance $Q_{0p} = 1/2$, and the quantities γ_{ra} and γ_{rb} are equal and contain the factor $1/2$ in comparison with γ_r for the appropriate single level (without phonons) coming through the crossing point of energy terms (see Fig.1).

VII. NON-RADIATIVE LIFETIMES OF MAGNETOPOLARON

We will not calculate the EHP inverse non-radiative lifetimes in a QW in a magnetic field far away from the magnetopolaron resonance as long as it is not clear what processes determine them. One-phonon transitions are forbidden by the energy conservation law. Perhaps the main contribution is determined by the two-phonon processes with acoustic phonons participation. But in the magnetophonon resonance vicinity *some contribution into* $\gamma_p(p = a, b)$, *appears due to the finite lifetimes of LO phonons composing the magnetopolaron*. We will calculate this contribution here and determine the low limit of $\gamma_p(p = a, b)$ in the resonance vicinity.

The phonon ν in a QW is characterized by the inverse non-radiative lifetime γ_ν due to phonon-phonon interaction, for instance, by the LO phonon decay into two acoustic phonons.

The quantity γ_ν may be written as

$$\gamma_\nu = \frac{2\pi}{\hbar} \sum_f |\langle f|\mathcal{H}_{ph-ph}|i\rangle|^2 \delta(E_i - E_f), \qquad (106)$$

where $\mathcal{H}_{ph-ph}$ is the phonon-phonon interaction, $|i\rangle = b_\nu^+|0\rangle$ is the initial state of the phonon ν with the energy $E_i = \hbar\omega_{LO}$, $|f\rangle = a_\tau^+|0\rangle$ is the phonon system final state with the set of indexes τ and the energy $E_f = E_\tau$, a_τ^+ is the phonon operator corresponding, for instance, to the creation of two acoustic phonons.

Taking into account that the component $\mathbf{q}_\perp$ in a QW is preserved in any transitions, we can write the matrix element in the RHS of Eq. (106) as

$$\langle 0|a_\tau \mathcal{H}_{ph-ph} b_\nu^+|0\rangle = \delta_{\mathbf{q}_\perp,\mathbf{q}'_\perp} V(\mathbf{q}_\perp, j, \vartheta), \qquad (107)$$

supposing that the initial state index set ν is

$$\nu \to \mathbf{q}_\perp, j, \qquad (108)$$

and the final state index set τ is

$$\tau \to \mathbf{q}'_\perp, \vartheta, \qquad (109)$$

where $\mathbf{q}'_\perp$ is the resultant transverse component of the wave vector in the final state, for example, the sum of the corresponding wave vectors of two phonons; $j(\vartheta)$ is the initial (final) state index set. Substituting Eq. (107) into Eq. (106) and summarizing on $\mathbf{q}_\perp$, we obtain

$$\gamma_\nu = \frac{2\pi}{\hbar} \sum_\vartheta |V(\mathbf{q}_\perp, j, \vartheta)|^2 \delta(\hbar\omega_{LO} - E_{\mathbf{q}_\perp,\vartheta}). \qquad (110)$$

Applying Eq. (106) we can now calculate the magnetopolaron inverse non-radiative lifetime $\gamma_{p,k_y,l}$ due to phonon-phonon interaction. The initial state wave function is written as $\Theta_{p,k_y,l}(\mathbf{r})|0\rangle$, where $|0\rangle$ is a phonon vacuum , the operator $\Theta_{p,k_y,l}(\mathbf{r})$ is determined in Eqs. (93) — (95). The final state wave function is the product of the electron and phonon wave functions

$$|f\rangle = \psi^e_{n',k'_y,l'}(\mathbf{r}) a_\tau^+|0\rangle, \qquad (111)$$

where $\psi^e_{n,k_y,l}(\mathbf{r})$ is determined in Eq. (66). Only the second term from the RHS of Eq. (93), describing the one LO phonon state, contributes into the matrix element $\langle f|\mathcal{H}_{ph-ph}|i\rangle$. The first term does not contribute because $\langle 0|a_\tau|0\rangle = 0$. One obtains

$$\langle f|\mathcal{H}_{ph-ph}|i\rangle = Q_{1p}^{1/2} \sum_\nu e^{ia_H^2 q_x(k_y - q_y/2)} \frac{U^*(\nu)}{|A|} \times \\ \times \int d\mathbf{r} \psi^{e*}_{n',k'_y,l'}(\mathbf{r}) \psi^e_{0,k_y-q_y,l}(\mathbf{r}) \times \\ \times \langle 0|a_\tau \mathcal{H}_{ph-ph} b_\nu^+|0\rangle. \qquad (112)$$

The initial (final) state is characterized by the indexes p, k_y, l $(n', k'_y, l', \mathbf{q}'_\perp, \vartheta)$. The index set ν in Eq. (112) consists of $\mathbf{q}_\perp$ and j. Taking into account the orthonormalization of the electron wave functions $\psi^e_{n,k_y,l}$ and Eq. (107) and summarizing in Eq. (112) on $\mathbf{q}_\perp$, one obtains

$$\langle f|\mathcal{H}_{ph-ph}|i\rangle = \frac{Q_{1p}^{1/2}}{|A|} \delta_{n',0}\delta_{l',l}\delta_{k'_y,k_y-q'_y} e^{ia_H^2 q'_x(k'_x + q'_y/2)} \times \\ \times \sum_j U^*(\mathbf{q}'_\perp, j) V(\mathbf{q}'_\perp, j, \vartheta). \qquad (113)$$

Substituting Eq. (113) into Eq. (106) and summarizing on indexes n', k'_y, l' of the final state we obtain

$$\gamma_p = \frac{2\pi}{\hbar} \frac{Q_{1p}}{A^2} \sum_{\mathbf{q}_\perp,\vartheta} \sum_{j_1,j_2} U^*(\mathbf{q}_\perp, j_1) U(\mathbf{q}_\perp, j_2) V(\mathbf{q}_\perp, j_1, \vartheta) \times \\ \times V^*(\mathbf{q}_\perp, j_2, \vartheta) \delta(E_p - \hbar\Omega_e/2 - E_{\mathbf{q}_\perp,\vartheta}). \qquad (114)$$

Eq. (114) is applicable for the model used in Ref. [26], where it was assumed that the confined and interface phonons, having the same non-dispersal frequency ω_{LO}, take part in the polaron creation. The index set j contains the phonon sort indexes.

It has been shown earlier [23] that interaction with the confined phonons in quite wide QWs may be approximately replaced by the Fröhlih interaction with the

bulk LO phonons and that interaction with the interface phonons does not contribute essentially into polaron energy splitting.

In the case of interaction with the bulk phonons the index j coincide with q_z, and Eq. (114) is simplified. One obtains instead of Eq. (107)

$$\langle 0|a_\tau \mathcal{H}_{ph-ph} b_\nu^+|0\rangle = \delta_{\mathbf{q},\mathbf{q}'} W(\mathbf{q},\varphi). \quad (115)$$

It is supposed that the phonon final state is characterized by the 3D wave vector $\mathbf{q}$ and by other indexes φ. With the help of Eq. (115) Eq. (114) is transformed into

$$\gamma_p = \frac{2\pi}{\hbar}\frac{Q_{1p}}{A^2}\sum_{\mathbf{q},\varphi}|U(\mathbf{q})|^2|W(\mathbf{q},\varphi)|^2 \times \\ \times \delta(E_p - \hbar\Omega_e/2 - E_{\mathbf{q},\varphi}). \quad (116)$$

Eq. (110) for the inverse LO phonon lifetime (if Eq. (115) is satisfied) can be written as

$$\gamma_{\mathbf{q}} = \frac{2\pi}{\hbar}\sum_{\varphi}|W(\mathbf{q},\varphi)|^2\delta(\hbar\omega_{LO} - E_{\mathbf{q},\varphi}), \quad (117)$$

where $E_{\mathbf{q},\varphi}$ is the final state energy.

Applying Eq. (96) for the polaron term energy E_p we obtain instead of Eq. (116)

$$\gamma_p = \frac{2\pi}{\hbar}\frac{Q_{1p}}{A^2}\sum_{\mathbf{q},\varphi}|U(\mathbf{q})|^2|W(\mathbf{q},\varphi)|^2 \times \\ \times \delta\left(\lambda/2 \pm \sqrt{(\lambda/2)^2 + A^2} + \hbar\omega_{LO} - E_{\mathbf{q},\varphi}\right), \quad (118)$$

where the upper (lower) sign corresponds to the term $a(b)$. Comparing Eq. (117) and Eq. (118) one finds that in the polaron resonance vicinity, i. e. under condition

$$\left|\lambda/2 \pm \sqrt{(\lambda/2)^2 + A^2}\right| << \hbar\omega_{LO}, \quad (119)$$

the quantity γ_p is expressed through $\gamma_{\mathbf{q}}$, i. e.

$$\gamma_p = Q_{1p}\bar{\gamma}_{LO}, \quad (120)$$

$$\bar{\gamma}_{LO} = \frac{\sum_{\mathbf{q}}\gamma_{\mathbf{q}}|U(\mathbf{q})|^2}{\sum_{\mathbf{q}}|U(\mathbf{q})|^2}. \quad (121)$$

Due to the factor Q_{1p} in Eq. (120) the quantity γ_p toughly depends on the parameter $\lambda = \hbar(\Omega_e - \omega_{LO})$. In the resonance

$$\gamma_p = \frac{1}{2}\bar{\gamma}_{LO} \quad (122)$$

for the both terms $p = a$ and $p = b$. One can suppose that the result from Eq. (114) is comparable with Eq. (120).

Let us consider the results of Eq. (118) far away from the resonance $\Omega_e = \omega_{LO}$. It is seen on Fig. 1 that the term a at $\lambda > 0$ and the term b at $\lambda < 0$ transits under condition $|\lambda| >> |A|$ into the electron level with indexes $n = 1, l$. According to Eq. (97) the factor Q_{1p} approximately equals to $Q_{1p} \simeq A^2/\lambda^2$. With the help of Eq. (118) one obtains approximately

$$\gamma_p = \frac{2\pi}{\hbar}\frac{A^2}{\lambda^2}\sum_{\mathbf{q},\varphi}\frac{|U(\mathbf{q})|^2}{A^2}|W(\mathbf{q},\varphi)|^2\delta(\hbar\Omega_e - E_{\mathbf{q},\varphi}). \quad (123)$$

This value is not expressed through $\bar{\gamma}_{LO}$, but one can see that in comparison with $\bar{\gamma}_{LO}$ it contains the small factor A^2/λ^2. Note that the argument of the δ-function in Eq. (123) appropriates to the transition $n = 1 \to n = 0$ with emitting, for instance, of two acoustic phonons. The quantity Eq. (123) is of the second order on phonon-phonon interaction. It is one of the contributions into the value of the EHP inverse non-radiative lifetime far away from the magnetophonon resonance. Two other branches of the terms, a at $\lambda < 0$ and b at $\lambda > 0, |\lambda| >> |A|$, appropriate to the state with the electron on the level $n = 0, l$ plus one LO phonon. $Q_{1p} \simeq 1$ on these branches, thus we obtain approximately

$$\gamma_p \simeq \bar{\gamma}_{LO}, \quad (124)$$

as it must be for a state including LO phonon. Thus the formula Eq. (118) gives the right limit transitions at $|\lambda| >> |A|$.

One can conclude that the quantity γ_p increases sharply around the electronic terms intersections reaching the half of the LO phonon inverse lifetime. *In the resonance $\Omega_e = \omega_{LO}$ the value of the inverse non-radiative lifetime for each of two terms is no smaller than $\bar{\gamma}_{LO}/2$.*

VIII. NUMERICAL CALCULATION RESULTS

The reflection $\mathcal{R}(\omega_l)$ and absorption $\mathcal{A}(\omega_l)$ coefficients for the three-level system and for the different interrelations between $\gamma_{r1(2)}$, $\gamma_{1(2)}$ and $\omega_1 - \omega_2$ are represented in Figs. 2 - 4. These results have to be used in the case of any two excited levels in a QW when they are situated quite close each other. When $\gamma_{r1(2)} << \omega_1 - \omega_2$, $\gamma_{1(2)} << \omega_1 - \omega_2$, the results for the two-level system are applicable.

Fig. 2 show the dependencies at $\gamma_{r1} = \gamma_{r2} = \gamma_r$, $\gamma_1 = \gamma_2 = \gamma$, $\gamma_r << \gamma$. The curves *1* are relevant to the case $\gamma < \omega_1 - \omega_2$, the curves *2* are relevant to the case $\gamma = \omega_1 - \omega_2$ and the curves *3* are relevant to the case $\gamma > \omega_1 - \omega_2$. It is seen that in case 1 the transit stems to the results for two-level systems. The maxima are situated in the vicinity of the points $\omega_l = \omega_1$ and $\omega_l = \omega_2$.

In Fig. 3 the functions $\mathcal{A}(\omega_l)$ and $\mathcal{R}(\omega_l)$ for the three level systems at $\gamma_{r1} = \gamma_{r2} = \gamma_r$, $\gamma_1 = \gamma_2 = \gamma$, $\gamma_r >> \gamma$ are represented. The curves *1* are relevant to the

case $\gamma_r < \omega_1 - \omega_2$, the curves *2* are relevant to the case $\gamma_r = \omega_1 - \omega_2$ and the curves *3-6* in Fig. 3a and the curves *3* in Fig. 3b are relevant to the case $\gamma_r > \omega_1 - \omega_2$. It is seen that in the case 1 the transit stems to the results for two two-level systems. In other cases the peculiar results can be seen. In particular the quantity $\mathcal{R}(\omega_0)$ approaches 0 on all the curves of Fig. 3b in the point $\omega_l = \omega_0$.

The reflection and absorption coefficients are much smaller than one and $\mathcal{R}(\omega_l) << \mathcal{A}(\omega_l)$ as in the case of the two-level systems at $\gamma_{r1(2)} << \gamma_{1(2)}$. On the contrary at $\gamma_{r1(2)} >> \gamma_{1(2)}$ $\mathcal{R}(\omega_l) >> \mathcal{A}(\omega_l)$ and $\mathcal{R}$ reaches one.

In Fig. 4 the functions $\mathcal{A}(\omega_l)$ and $\mathcal{R}(\omega_l)$ for the three-level systems at $\gamma_{r1} = \gamma_{r2} = \gamma_r, \quad \gamma_1 = \gamma_2 = \gamma, \quad \gamma_r = \gamma$ are represented. The curves *1* are relevant to the case $\gamma_r < \omega_1 - \omega_2$, the curves *2* are relevant to the case $\gamma_r = \omega_1 - \omega_2$ and the curves *3* are relevant to the case $\gamma_r > \omega_1 - \omega_2$. It is seen that the absorption coefficient reaches the maximum values but does not exceed 0.5.

In Fig. 5 the energy terms a and b for the system consisting of the polaron A (Fig. 1) and the hole with indexes $n = 1, l$ are represented. ΔE is term's splitting at $\lambda = 0$. $\mathcal{E}_p$ is the system energy measured from

$$E_0 = E_g + \varepsilon_l^e + \varepsilon_l^h + (3/2)(1 + m_e/m_h)\hbar\omega_{LO}.$$

According to Eq. (102) and Eq. (96),

$$\frac{\mathcal{E}_p}{\Delta E} = \frac{\lambda}{\Delta E}\left(1 + \frac{3}{2}\frac{m_e}{m_h}\right) \pm \frac{1}{2}\sqrt{1 + \left(\frac{\lambda}{\Delta E}\right)^2}, \quad (125)$$

where the upper (lower) sign corresponds to the term $a(b)$. The value $m_e/m_h = 0.2$ is applied, which is relevant to GaAs. Calculating $\mathcal{A}(\omega_l)$ and $\mathcal{R}(\omega_l)$ for magnetopolarons in QWs one has to substitute the quantities $\mathcal{E}_a$ and $\mathcal{E}_b$ (see Eq. (125)) into all the formulae of the section IY as energies $\hbar\omega_1$ and $\hbar\omega_2$ when $\hbar\omega_l$ is measured from the level E_0.

In Fig. 6 the dependencies γ_{rp} and γ_p/γ_{rp} from the magnetic field value are represented. Fig. 9 (10) appropriates to the term a (b). The constant γ_0 is the EHP inverse radiative lifetime at $\lambda = 0$ without the polaron effect. According to the section Y,

$$\gamma_0 = \frac{2e^2\omega_{LO}m_e}{\hbar cnm_0}\frac{p_{cv}^2}{m_0\tilde{E}_g}\pi_{ll}^2. \quad (126)$$

With the help of Eq. (105) we obtain

$$\frac{\gamma_{rp}}{\gamma_0} = \frac{1}{2}\left(1 + \frac{\lambda/\Delta E}{\Delta E/\hbar\omega_{LO}}\right)\left[1 \pm \frac{\lambda/\Delta E}{\sqrt{1 + (\lambda/\Delta E)^2}}\right], \quad (127)$$

which was applied in Fig. 6. The upper (lower) sign is relevant to the term $p = a$ (b). The factor in the parenthesis in the RHS of Eq. (127) describes the magnetic field dependence of Ω_0 entering in γ_{rp} (see Eq. (105)). This factor depends on $\Delta E/\hbar\omega_{LO}$, which is chosen equal to 0.18. **

According to Eq. (105) and Eq. (120)

$$\frac{\gamma_p}{\gamma_{rp}} = \frac{\bar{\gamma}_{LO}/\gamma_0}{1 + \frac{\lambda/\Delta E}{\Delta E/\hbar\omega_{LO}}}\frac{1 \mp \frac{\lambda/\Delta E}{\sqrt{1+(\lambda/\Delta E)^2}}}{1 \pm \frac{\lambda/\Delta E}{\sqrt{1+(\lambda/\Delta E)^2}}}, \quad (128)$$

where the upper (lower) sign appropriates to the term $p = a$ (b).

Eq. (127) and Eq. (128) were applied in Figs. 9, 10. On Figs. 9, 10 the function γ_p/γ_{rp} for the three values of $\bar{\gamma}_{LO}/\gamma_0$ is represented: 10 (curve *1*), 1 (curve *2*), 0.1 (curve *3*). According to our estimate with applying the parameter values from Ref. [32]

$$\hbar\gamma_0 \simeq 5.35 \cdot 10^{-5} eV. \quad (129)$$

Eqs. (125) - (128) and Fig. 5 - 6 allow us in principle to determine $\mathcal{A}(\omega_l)$ and $\mathcal{R}(\omega_l)$ with the help of the results of the section IY at a fixed value of magnetic field. One can also determine $\mathcal{A}(H)$ and $\mathcal{R}(H)$ at fixed ω_l values.

The quantity $\bar{\gamma}_{LO}$ remains practically unknown. †† Therefore on Figs. 6 - 8 the results for the different interrelations $\bar{\gamma}_{LO}/\gamma_0$ are represented.

One can conclude from Fig. 5 that the magnetopolaron effect area spreads approximately from $\lambda/\Delta E = -2$ to $\lambda/\Delta E = 2$. At $\lambda/\Delta E > 2$ the term a (and at $\lambda/\Delta E < -2$ the term b) transits into the EHP energy level with quantum numbers of the electron and hole $n_e = n_h = 1, \quad l_e = l_h = l$. The term b at $\lambda/\Delta E > 2$ and the term a at $\lambda/\Delta E < -2$ transit into the energy level of the system consisting of the EHP with quantum numbers $n_e = 0, n_h = 1, \quad l_e = l_h = l$ and one LO phonon. This state interacts weakly with exciting light out of the magnetophonon resonance.

In the case of the polaron A the value ΔE of energy splitting is very large. Therefore the conditions $\gamma_{rp} << \omega_1 - \omega_2, \quad \gamma_p << \omega_1 - \omega_2$ are obviously satisfied and the approximation of two two-level systems is applicable for calculations of $\mathcal{A}$ and $\mathcal{R}$, i. e. Eq. (36) and Eq. (37) of the section IY. An application of the results of the same section, relevant to the three-level systems, is necessary in the case of smaller values ΔE, which would observed, for instance, in the case of weak polarons (see the section II).

In the magnetopolaron area, two maxima have to be observed in the functions $\mathcal{A}(\omega_l)$ and $\mathcal{R}(\omega_l)$ at the definite

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** According to Ref. [23] in GaAs (the polaron A) at $d = 300A$ energy splitting is $\Delta E \simeq 6.6510^{-3} eV$. As long as in GaAs at $\hbar\omega_{LO} = 0.0367eV$ one obtains $\Delta E/\hbar\omega_{LO} \simeq 0.181$.

†† One can try to estimate $\bar{\gamma}_{LO}$ on the line width of one-phonon scattering in the bulk GaAs.

value H of the magnetic field. The maximum points ω_{la} and ω_{lb} one can determine , drawing the vertical sections on Fig. 5. Two maxima have to be observed also in functions $\mathcal{A}(H)$ and $\mathcal{R}(H)$ at $\omega_l = const$. The appropriate magnetic field values H_a and H_b can be determined drawing the horizontal sections on Fig. 5. The quantities γ_{rp} and γ_p are strongly dependent on magnetic field in the magnetophonon resonance vicinity (see Fig. 6).

The results for $\mathcal{A}(\omega_l)$ and $\mathcal{R}(\omega_l)$ are different for the different values of $\bar{\gamma}_{LO}/\gamma_0$. At those values of $\lambda/\Delta E$ (i.e. the magnetic fields H), where $\gamma_p/\gamma_{rp} >> 1$, light absorption is much smaller than one , but exceeds light reflection. In the case of curves *1* on Figs. 9,10, relevant to the parameter $\bar{\gamma}_{LO}/\gamma_0 = 10$, this area for the both terms a and b takes almost the whole interval of magnetic field values where the polaron effect is essential. Indeed, for the curves *1* in Fig. 6 $\gamma_a/\gamma_{ra} = 1$ at $\lambda_{a1}/\Delta E = 1.24$, and $\gamma_b/\gamma_{rb} = 1$ at $\lambda_{b1}/\Delta E = -1.82$. Around the points λ_{a1} and λ_{b1} light reflection by terms a and b reaches the largest values ($\mathcal{A}$=1/2 and $\mathcal{R}$=1/4 in maximum). In the regions $\lambda >> \lambda_{a1}$, and $\lambda << \lambda_{b1}$ the polaron effect is inessential. The second maxima on the curves $A(\omega_l)$ and $R(\omega_l)$ disappear. These results are alike the results for the EHP with $n_e = n_h = 1, \quad l_e = l_h = l$ far from the polaron resonance. In this area $\gamma_p/\gamma_{rp} << 1$, and light reflection exceeds light absorption.

In the opposite case $\bar{\gamma}_{LO}/\gamma_0 = 0.1$ (curves *3*) the condition $\gamma_p/\gamma_{rp} << 1$ is satisfied in almost the whole region of the magnetophonon resonance.

This region includes the resonant point $\lambda = 0$. Light absorption is much smaller than light reflection which reaches the values $\mathcal{R} = 1$ in both maxima on the curve $\mathcal{R}(\omega_l)$. At $\lambda_{a3}/\Delta E = -1.24$ and $\lambda_{b3}/\Delta E = 1.68$ the inverse non-radiative and radiative lifetimes become equal , which corresponds to maximum absorption. On the left of the point λ_{a3} the term a does not interact with light (note the small values γ_{ra}). The same is relevant to the term b on the right of λ_{b3}.

The curves *2* on Figs. 6 are relevant to an intermediate case $\bar{\gamma}_{LO}/\gamma_0 = 1$. The maximum absorption point coincides with the resonance point $\lambda = 0$. For the term a at $\lambda > 0$ the condition $\gamma_a/\gamma_{ra} < 1$ is satisfied, at $\lambda < 0$ the opposite condition $\gamma_a/\gamma_{ra} > 1$ is satisfied. There is the opposite picture for the term b. At $\gamma_p/\gamma_{rp} < 1$ light reflection dominates, at $\gamma_p/\gamma_{rp} > 1$ light absorption dominates .

Figs. 7, 8 $\mathcal{A}(H)$ and $\mathcal{R}(H)$ in the polaron splitting vicinity. The curves *1, 2, 3* correspond to three different magnitudes of the frequency ω_l. The Figs. 7a and 8a correspond to the ratio $\bar{\gamma}_{LO}/\gamma_0 = 10$. The Figs. 7b and 8b correspond to the ratio $\bar{\gamma}_{LO}/\gamma_0 = 0.4$.

To summarize, a classification of magnetopolarons in a QW in strong magnetic field has been done. The usual polarons (including double-, triple- and so on), combined polarons (for which the resonant magnetic field depends on the QW width and depth) as well as the weak polarons (for which a polaron energy splitting ΔE is comparatively small) have been defined.

The formulae for the non-dimensional coefficients of light absorption and reflection by QW under normal incidence of light have been obtained for the three-level electronic systems. Our calculations are based on taking into account the radiative lifetimes of the electronic systems on two excited level, which means taking into account all of the absorption and reradiation processes and a transcendence of the perturbation theory on the light-electron coupling constant.

The EHP radiative lifetime in a QW in a strong magnetic field under an arbitrary value of the QW inplane EHP wave vector $\mathbf{K}_\perp$ has been calculated far from the magnetophonon resonance. The radiative lifetimes of a system, consisting of a magnetopolaron and a hole, have been calculated.

The expressions for the contributions into non-radiative lifetimes of the polaron states due to the finite LO phonon lifetimes are obtained. It turned out that the radiative lifetimes of polarons as well as non-radiative ones are strongly dependent on magnetic field H in the magnetopolaron resonance vicinity.

The functions $\mathcal{A}(H)$ and $\mathcal{R}(H)$ have been obtained by taking into account the magnetopolaron effect.

IX. ACKNOWLEDGEMENTS

S.T.P thanks the Zacatecas University and the National Council of Science and Technology (CONACYT) of Mexico for the financial support and hospitality. D.A.C.S. thanks CONACYT (27736-E) for the financial support. Authors are grateful to V. M. Bañuelos-Alvarez for the computer cooperation and to A. D'Amor for a critical reading of the manuscript. This work has been partially supported by the Russian Foundation for Basic Research and by the Program "Solid State Nanostructures Physics".

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FIG. 1. The energy levels of an electron (hole)–LO phonon system as a function of the cyclotron frequency Ω. E is the energy measured from the size-quantization energy level ε. The double, triple, quaternate, weak and combined polarons are marked with the filled circles, triangles, squares, empty circles and crossed circles, respectively.

FIG. 2. Non-dimensional light absorption $\mathcal{A}$ and reflection $\mathcal{R}$ coefficients of a three-level system as a function of the light frequency ω_l for the case $\gamma >> \gamma_r$, where $\gamma = \gamma_1 = \gamma_2$ is the non-radiative homogeneous level broadening, $\gamma_r = \gamma_{r1} = \gamma_{r2}$ is the radiative level broadening. Parameter magnitudes have been used: $\hbar(\omega_1 - \omega_2) = 0.005$, $\hbar\gamma_r = 0.0001$, $\hbar\gamma = 0.002$ (curve *1*), $\hbar\gamma = 0.005$ (curve *2*) $\hbar\gamma = 0.008$ (curve *3*) in eV or in arbitrary units.

FIG. 3. Same as Fig. 2 for the case $\gamma << \gamma_r$. The parameter magnitudes have been used: $\hbar(\omega_1 - \omega_2) = 0.005$, $\hbar\gamma = 0.0001$. Fig. 3a: $\hbar\gamma_r = 0.002$ (curve *1*) , $\hbar\gamma_r = 0.005$ (curve *2*) and $\hbar\gamma_r = 0.008$ (curve *3*). On the inset: $\hbar\gamma_r = 0.04$ (curve *4*), $\hbar\gamma_r = 0.125$ (curve *5*) and $\hbar\gamma_r = 0.5$ (curve *6*). Fig. 3b: $\hbar\gamma_r = 0.002$ (curve *1*), $\hbar\gamma_r = 0.005$ (curve *2*) and $\hbar\gamma_r = 0.008$ (curve *3*).

FIG. 4. Same as Fig. 2 for the case $\gamma = \gamma_r$. The absorption (a) and reflection (b) coefficients. The parameter magnitudes have been used: $\hbar(\omega_1 - \omega_2) = 0.005$, $\hbar\gamma = \hbar\gamma_r = 0.002$ (curve *1*); $\hbar\gamma = \hbar\gamma_r = 0.005$ (curve *2*), $\hbar\gamma = \hbar\gamma_r = 0.008$ (curve *3*).

FIG. 5. Excitation energies of an electron system consisting of a type-A polaron (see Fig. 1) and a hole with quantum numbers $n_h = 1$, l as a function of magnetic field. The parameter $m_e/m_h = 0.2$ (corresponding to GaAs) has been used. The energy $\mathcal{E}_p$ is measured from the level $E_0 = E_g + \varepsilon_l^e + \varepsilon_l^h + (3/2)(1 + m_e/m_h)\hbar\omega_{LO}, \Delta E$ is the polaron energy splitting in the point $\Omega_e = \omega_{LO}$.

FIG. 6. The radiative and non-radiative inverse lifetimes of electronic excitations in a QW consisting of a polaron A and a hole with quantum numbers $n_h = 1$ and l as a function of magnetic field. The parameter $\Delta E/\hbar\omega_{LO} = 0.18$ (corresponding to GaAs, Ref. [23]) has been used. γ_{rp}/γ_0 (solid lines), γ_p/γ_{rp} (dashed lines). The index p designates a and b. Fig. 6a(b) appropriates to $p = a(b)$. The parameter magnitudes have been used for the dashed lines: $\bar{\gamma}_{LO}/\gamma_0 = 10$ (curve *1*), $\bar{\gamma}_{LO}/\gamma_0 = 1$ (curve *2*) and $\bar{\gamma}_{LO}/\gamma_0 = 0.1$ (curve *3*).

FIG. 7. The light absorption coefficient $\mathcal{A}$ in a polaron splitting vicinity as a function of magnetic field. The polaron A (see Fig. 1) parameters for GaAs have been used: $\Delta E = 6.65 \cdot 10^{-3}$ eV (Ref. 23), $\gamma_0 = 10^{-4} eV$. a corresponds to $\bar{\gamma}_{LO}/\gamma_0 = 10$, b corresponds to $\bar{\gamma}_{LO}/\gamma_0 = 0.4$. The curves *1*, *2* and *3* distinguish with different values of ω_l. $\Delta\omega_l = \omega_l - (\varepsilon_a + \varepsilon_b)/2\hbar = 0$ (curves *1*), $\Delta\omega_l = \Delta E/2$ (curves *2*) and $\Delta\omega_l = -\Delta E/2$ (curves *3*).

FIG. 8. The light reflection coefficient R in the polaron splitting vicinity as a function of magnetic field. Parameters and designations of Fig. 7 have been used.